A comparison of ***latent variable*** (*g*-Factor) and ***mutualistic network models*** using ***longitudinal data***: *An application of learning mathematics*

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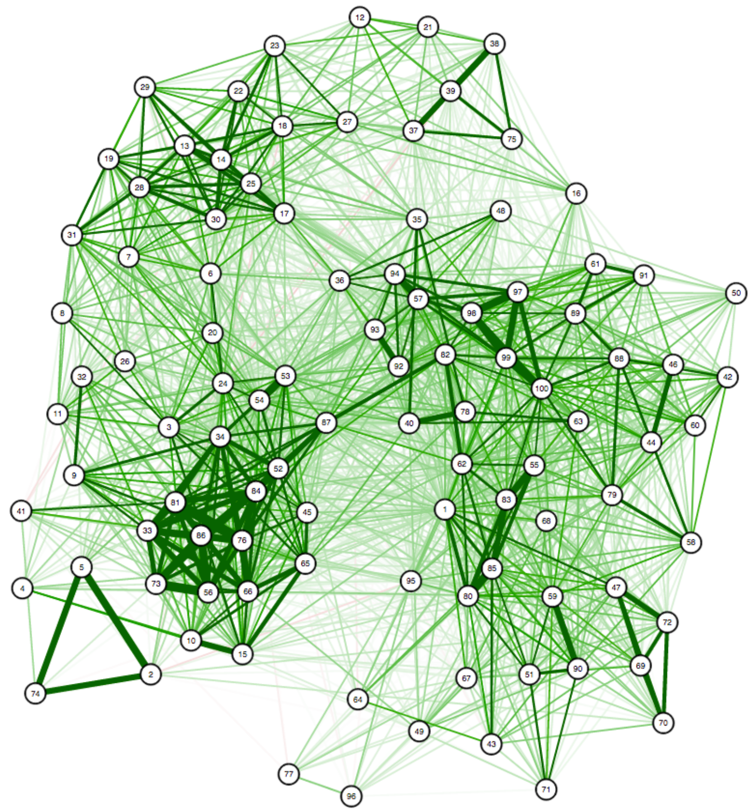
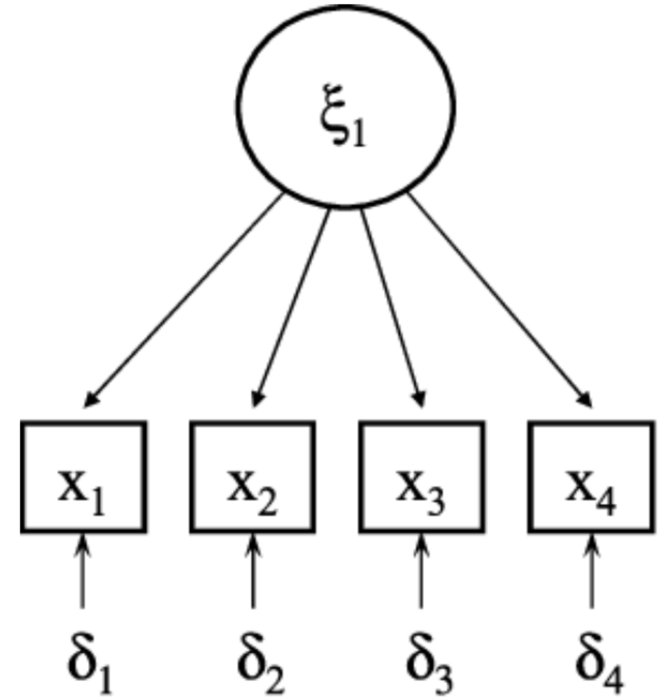
Workshop:

USING PSYCHOMETRICS TO IMPROVE COGNITIVE MODELS–AND THEORY

COGSCI - 2024

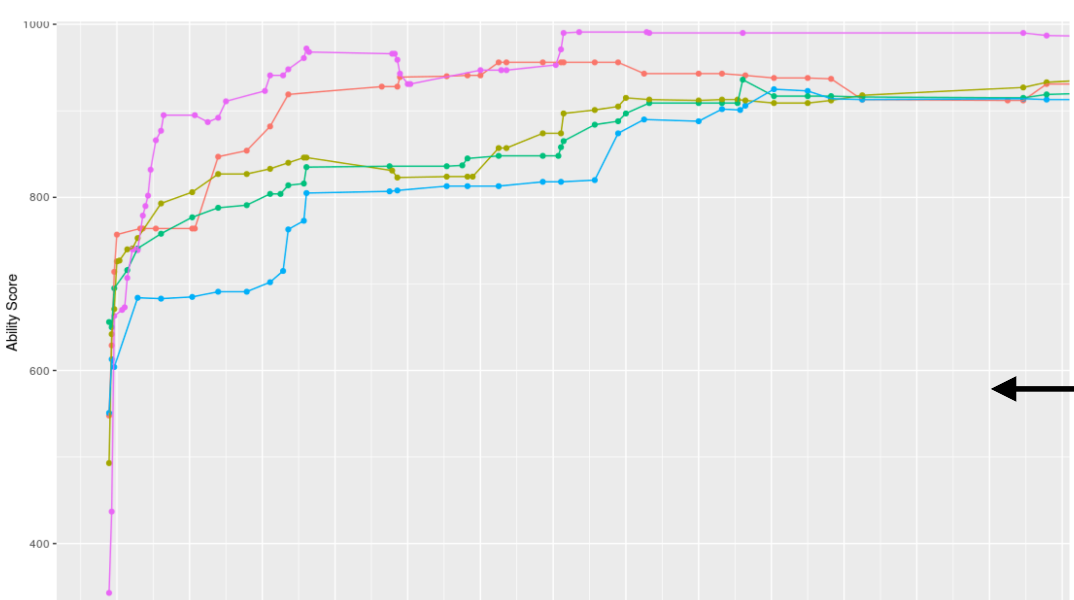
Positive manifold





Factor Analysis

Developmental data

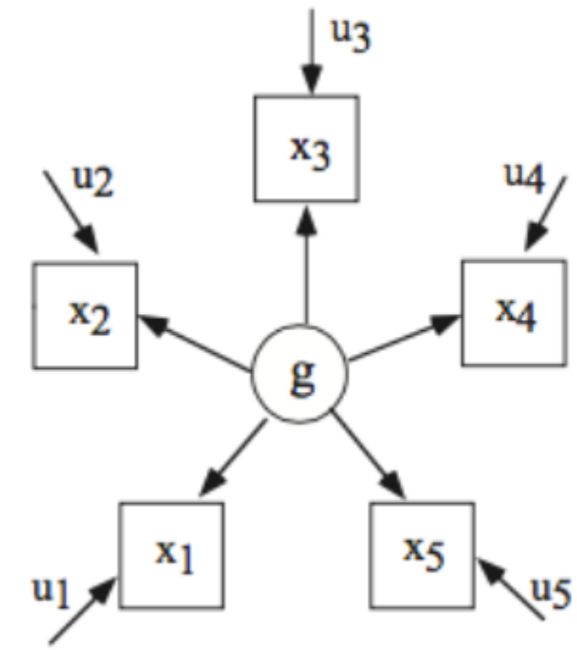


Ontology:
Causal construct vs index (e.g., Borsboom[1]; Van der Maas[2])

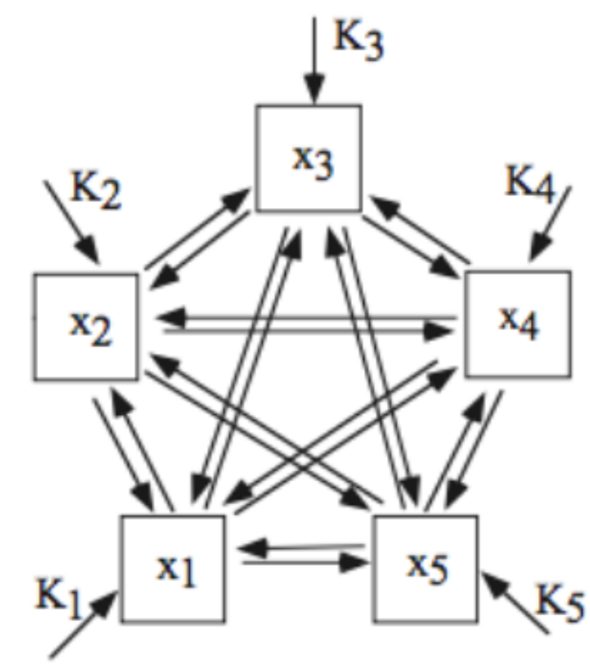
What is the true data generating mechanism? (e.g., Kruis [3])

Compare Two Explanations | *Predictions*

- *g*-Factor (Spearman [4]):
  - Development is caused by changes in the true ability *g*



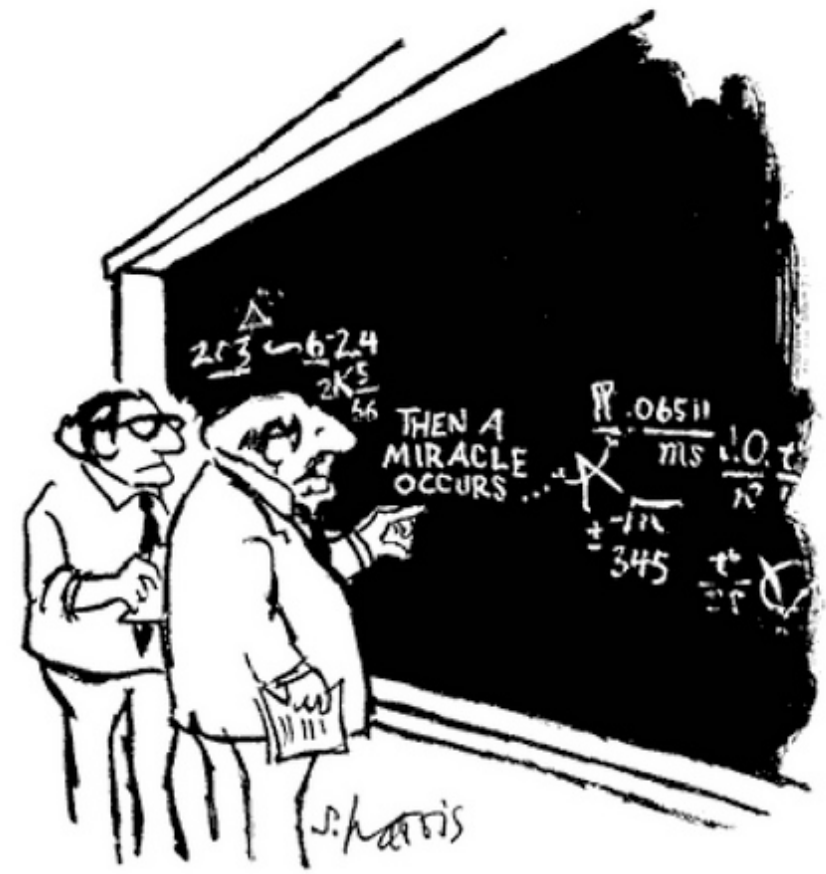
- Mutualism (Van der Maas [5])
  - Development is caused by a dynamical model including reciprocal causation or mutualism.



Compare Two Explanations | *Implications*

- *g*-Factor
  - *g* exists independent of the collected data and has an **causal role** in the data generating system
  - Were is *g* located? Can we (ultimately) uncover the latent aspect and truly observe *g*?

- Mutualism
  - *g* is an emerging property of the dynamical system that drives development and **does not** have any interpretation more than an '**index**' variable (data reduction)
  - What are the wiring mechanisms of the developmental system (edges)? Where are individual differences present?



"I think you should be more explicit here in step two."

Compare Two Explanations | *Modelling Framework*

"When thinking about any repeated measures analysis it is best to ask first, what is your model for change?" (McArdle [6], p 579)

Latent Change Score Models

1. Structural equation models aimed at measuring (predicting) changes between time-points

2. + Developmental dataset, no assumptions of stationarity

3. + Predicting changes and not mean scores

Methods | *Latent Change Score Models*

Regression model: $$y_{pt} = \beta_{t,t-1} \times y_{tp-1} + \Delta_{tp}$$

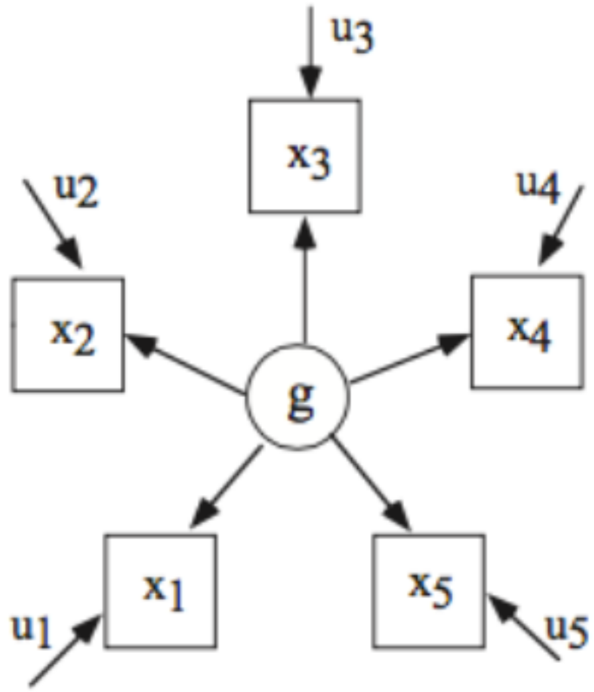
$\beta_{t,t+1} = 0$: $$\Delta_{pt} = y_{pt} - y_{pt-1}$$

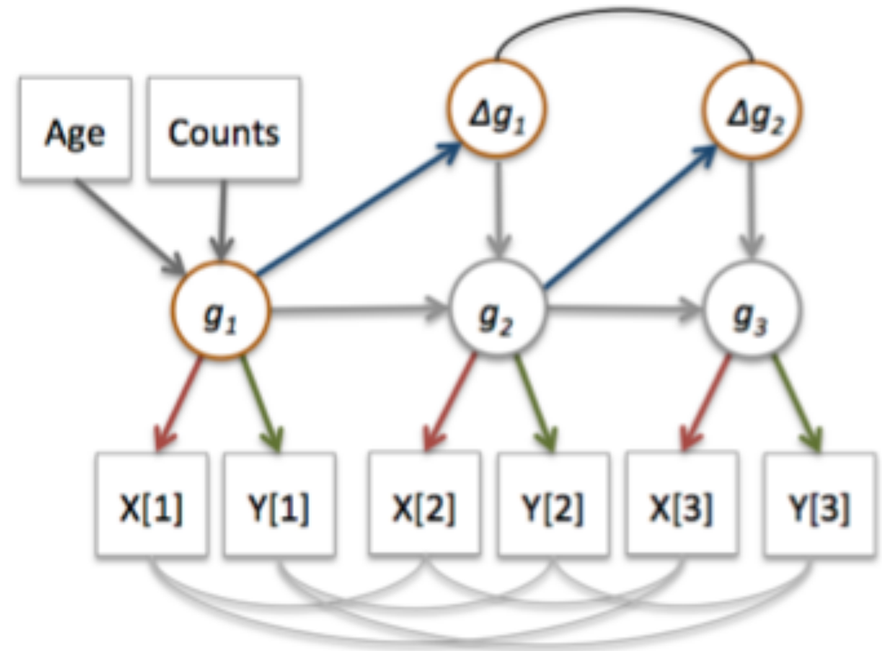
Bivariate extension:
$$\Delta_{1,pt} = \beta_1 y_{1,pt-1} + \gamma_{21} y_{2,pt-1}$$
$$\Delta_{2,pt} = \beta_2 y_{2,pt-1} + \gamma_{12} y_{1,pt-1}$$

β = self-feedback; **γ** = coupling

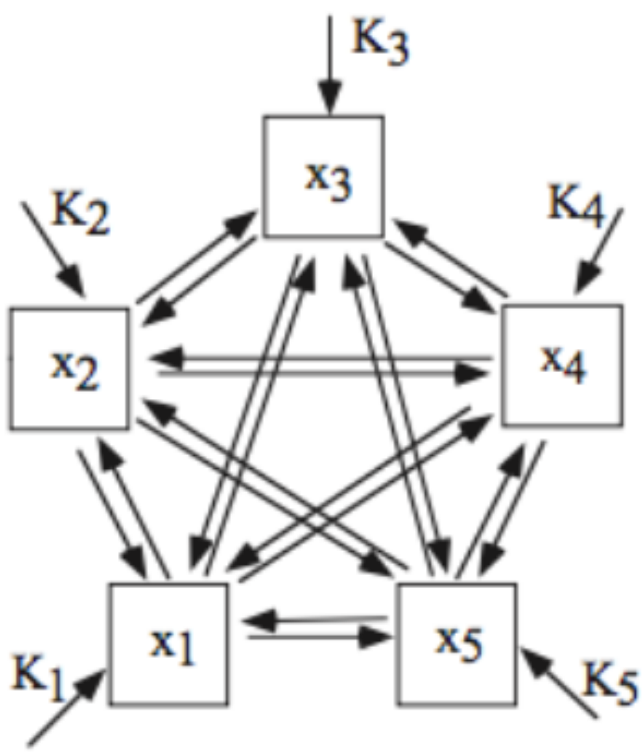
Methods | *Latent Change Score Models*

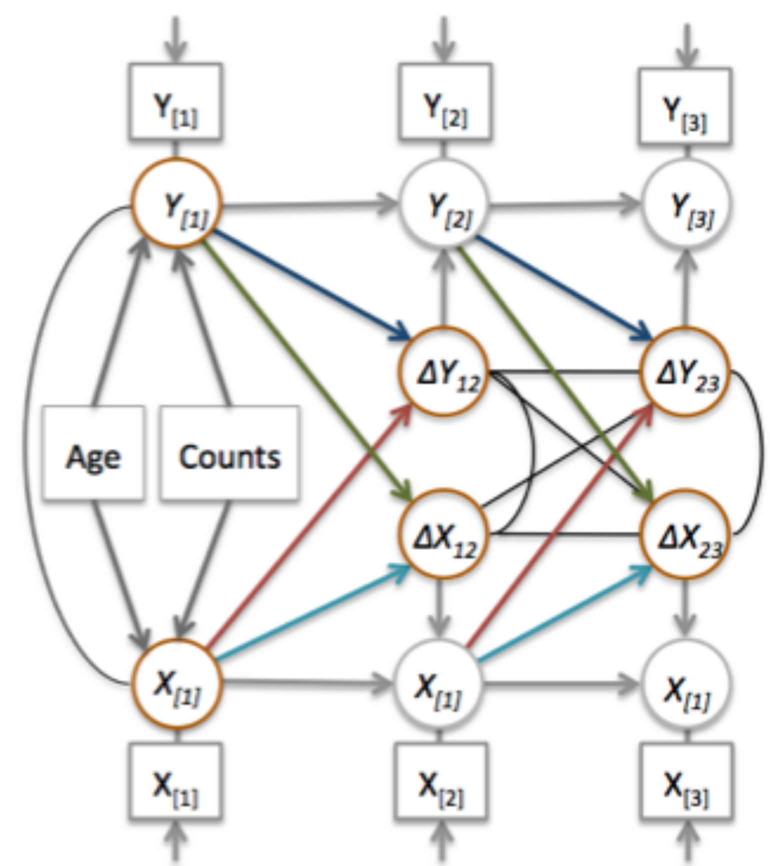
Model 1: *g*-factor





Model 2: *mutualism*





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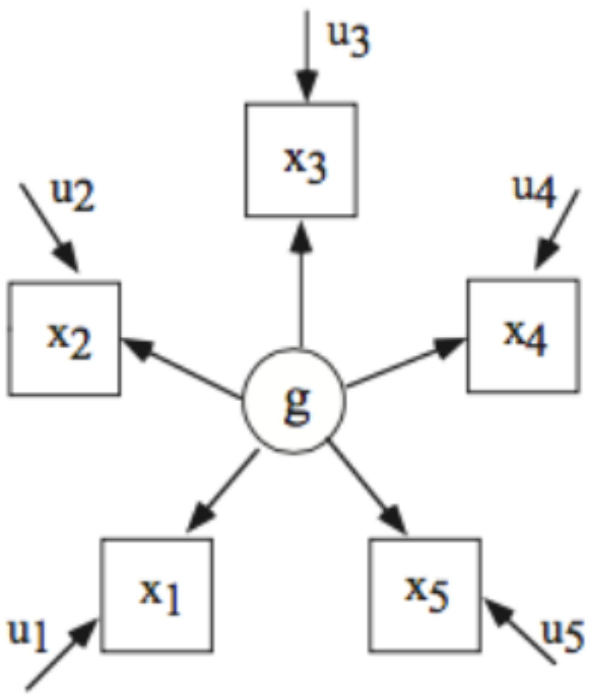

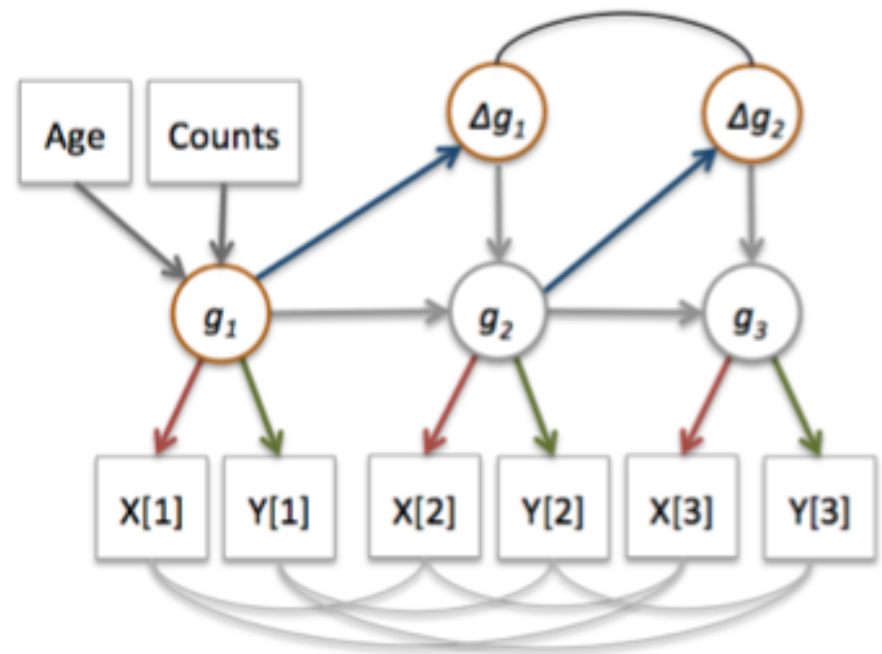

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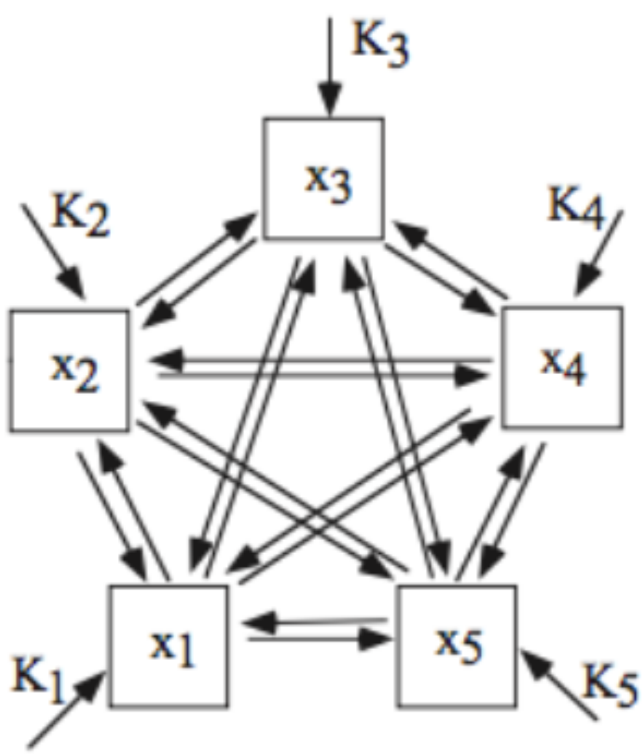

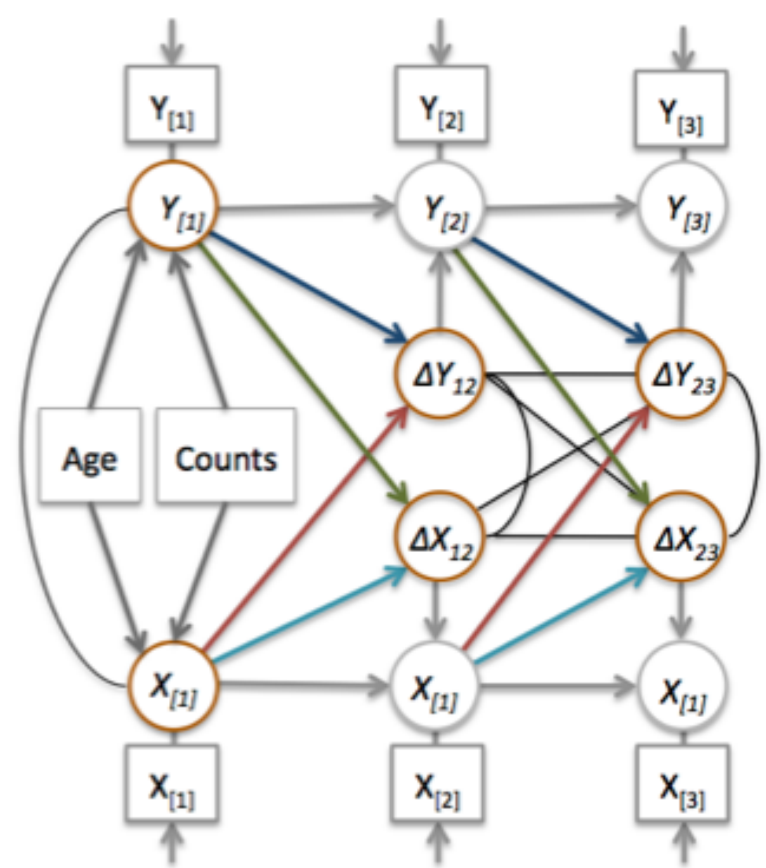

Model 3:

no coupling model

(coupling = 0)

serves as a baseline model

Methods | *Latent Change Score Models*

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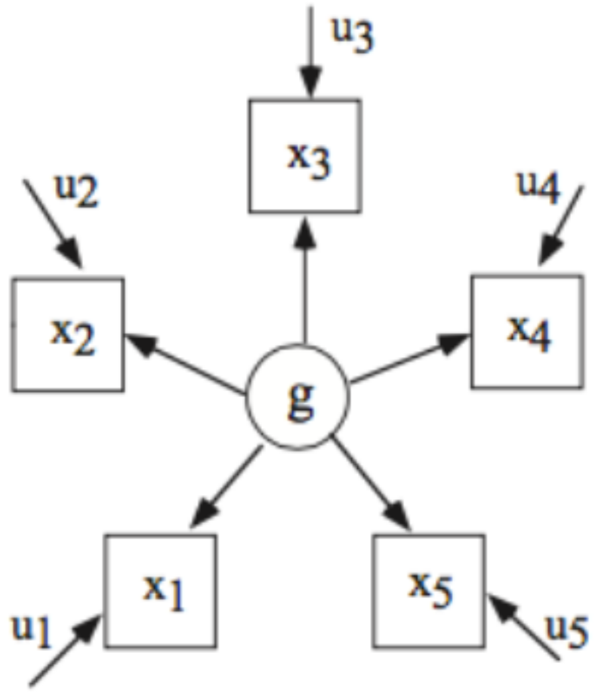

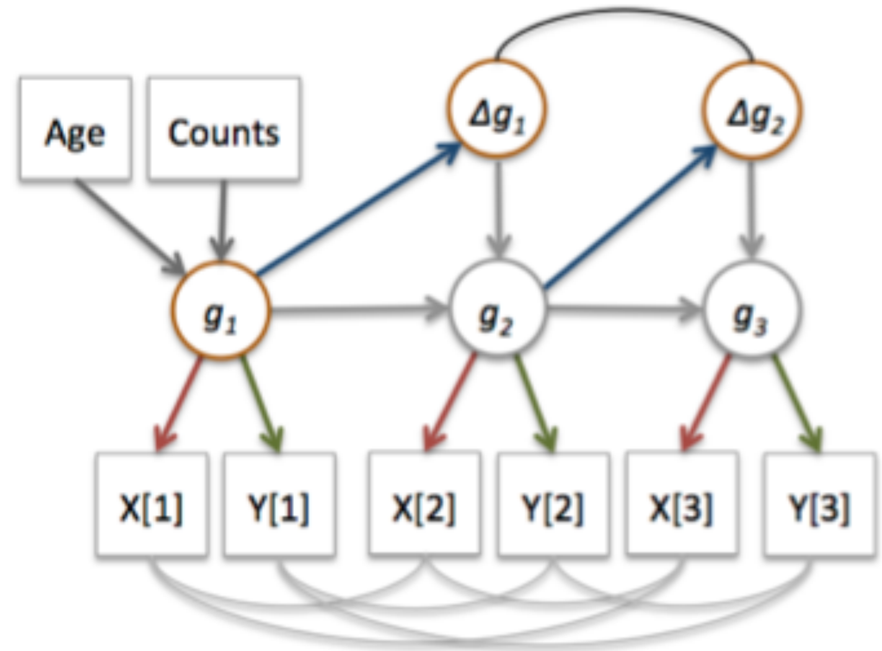

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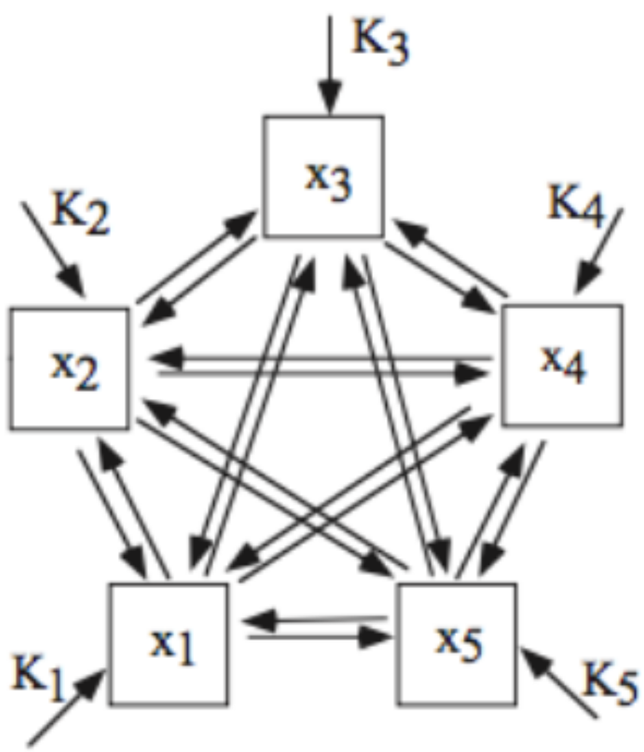

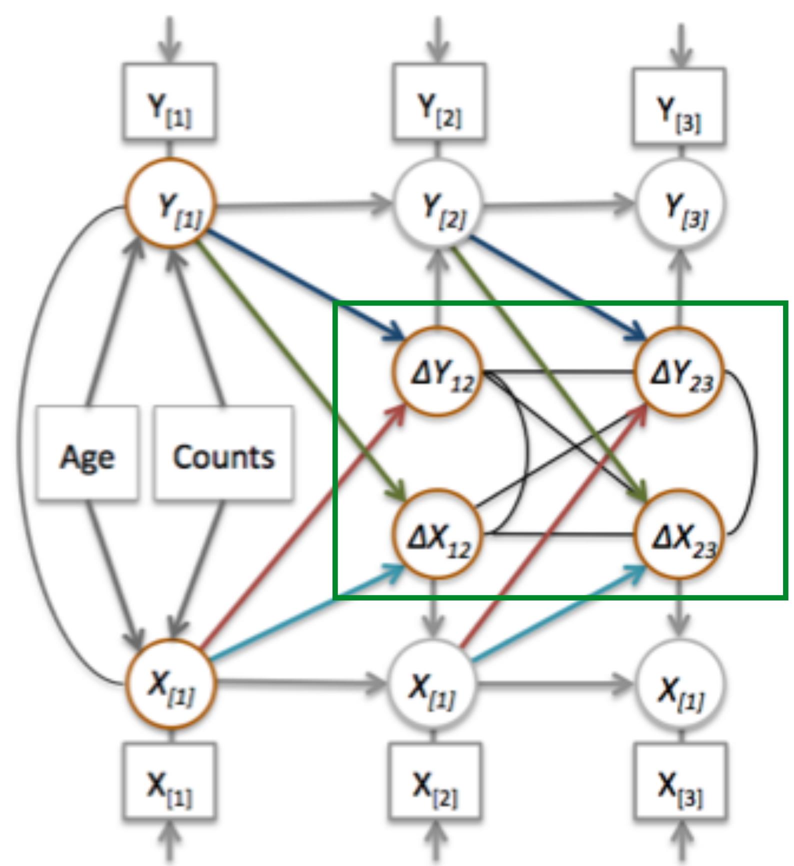

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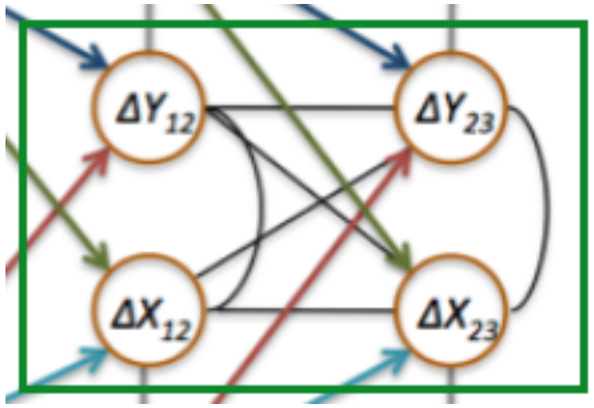
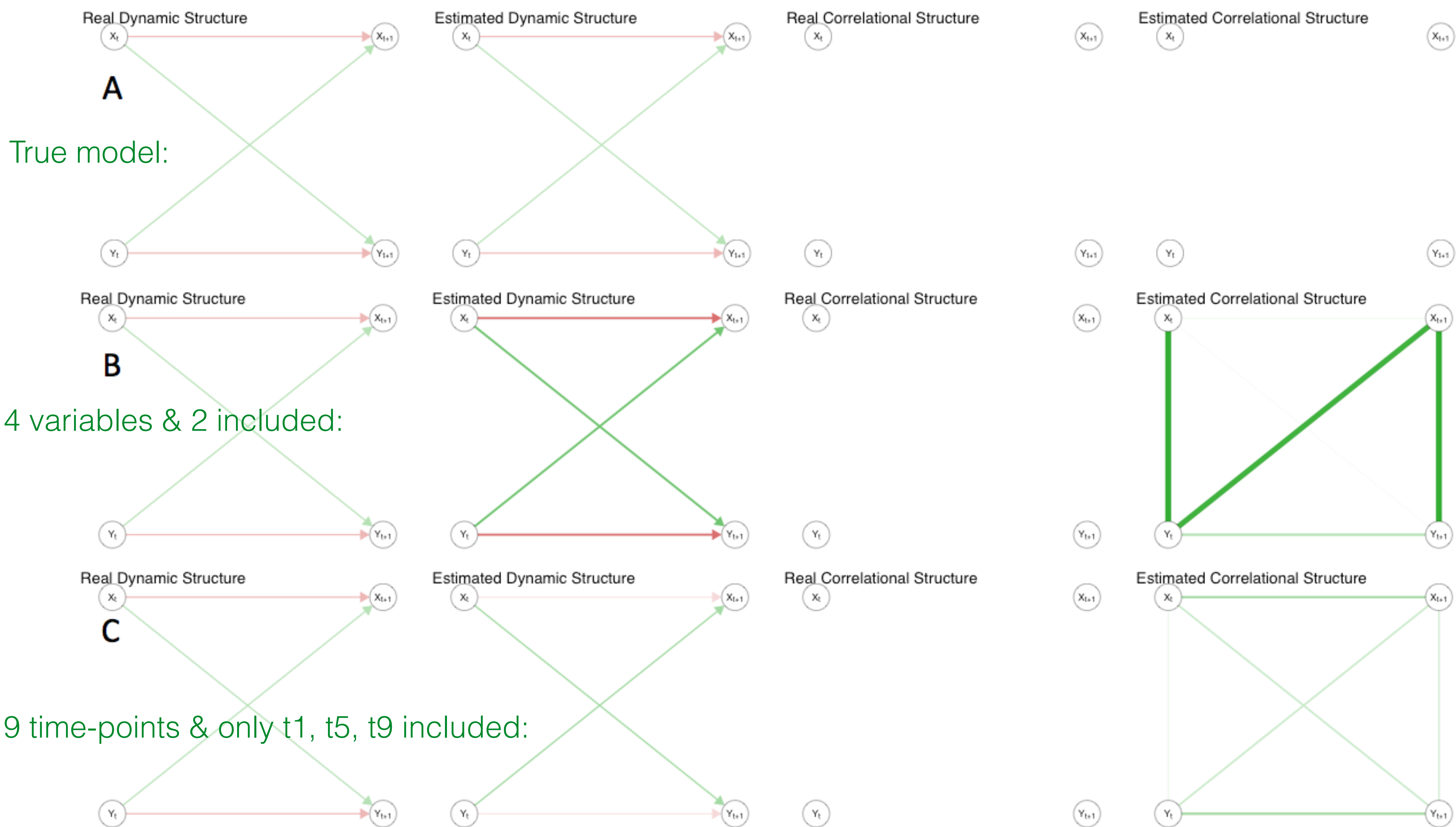
Methods | *Correlated Change Scores*

- *g*-Factor
  - Change scores should be correlated, since *g* drives the changes in multiple domains.
- Mutualism
  - In principle change scores should be uncorrelated. *But if:*
    - *a subset of all variables in the dynamical system are observed (Scenario B), or:*
    - *a part of all time-points are observed (Scenario C), than :*
  - *these correlations are inflated.*



Methods | *Correlated Change Scores*

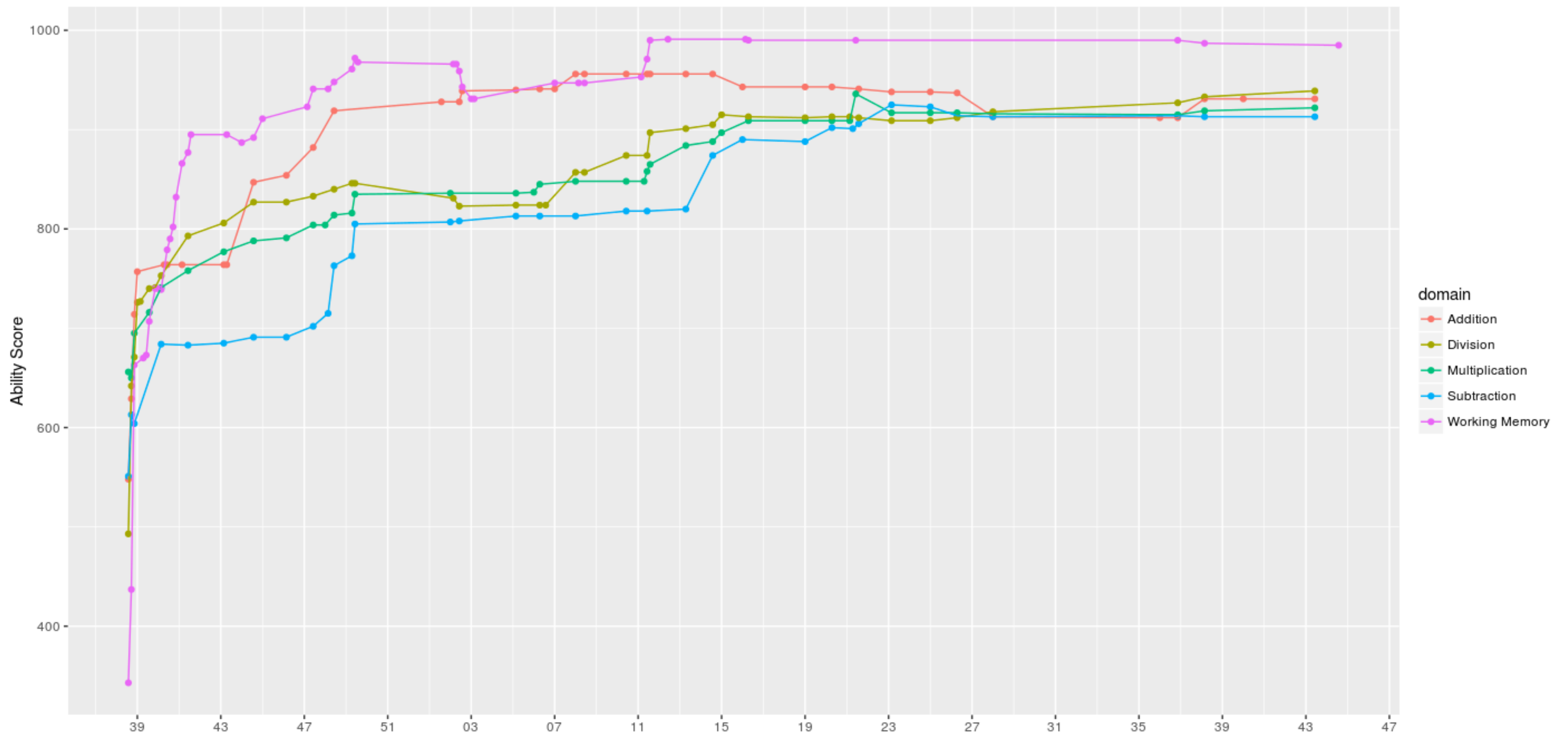


Methods | *Data: Math Garden*

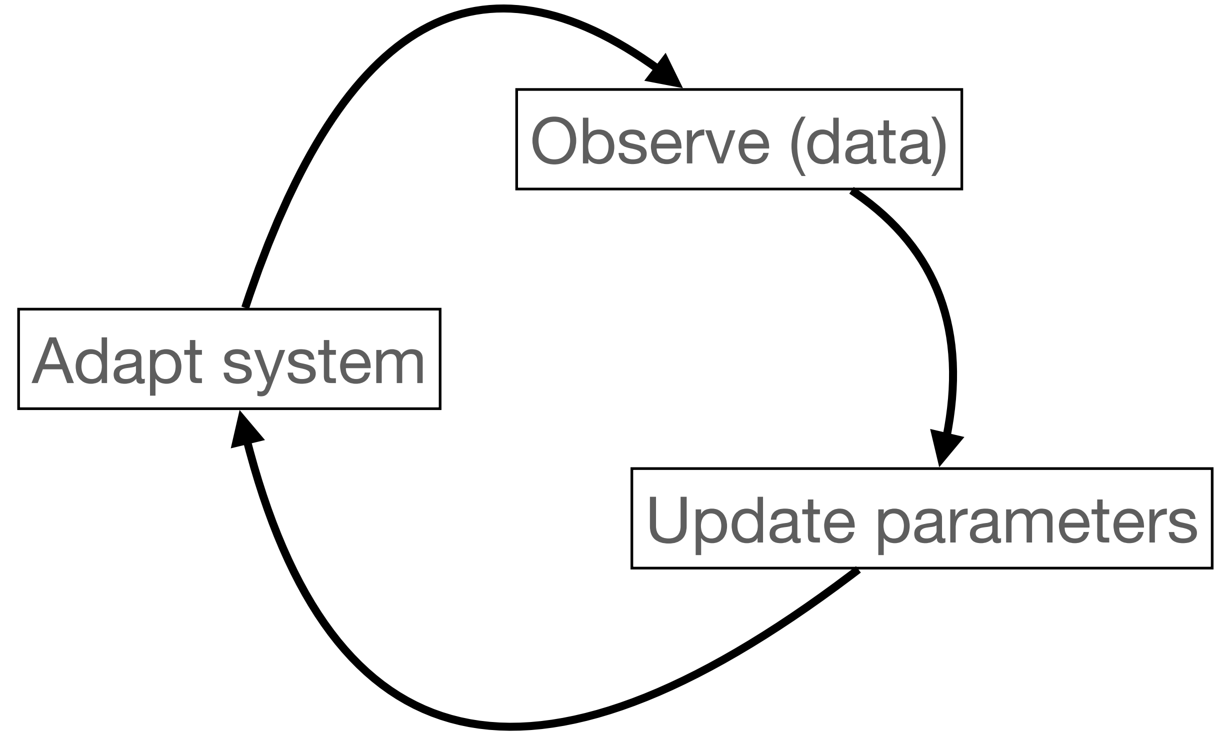
- **Online adaptive learning program** of **maths** aimed to collect large time-intensive data to study learning

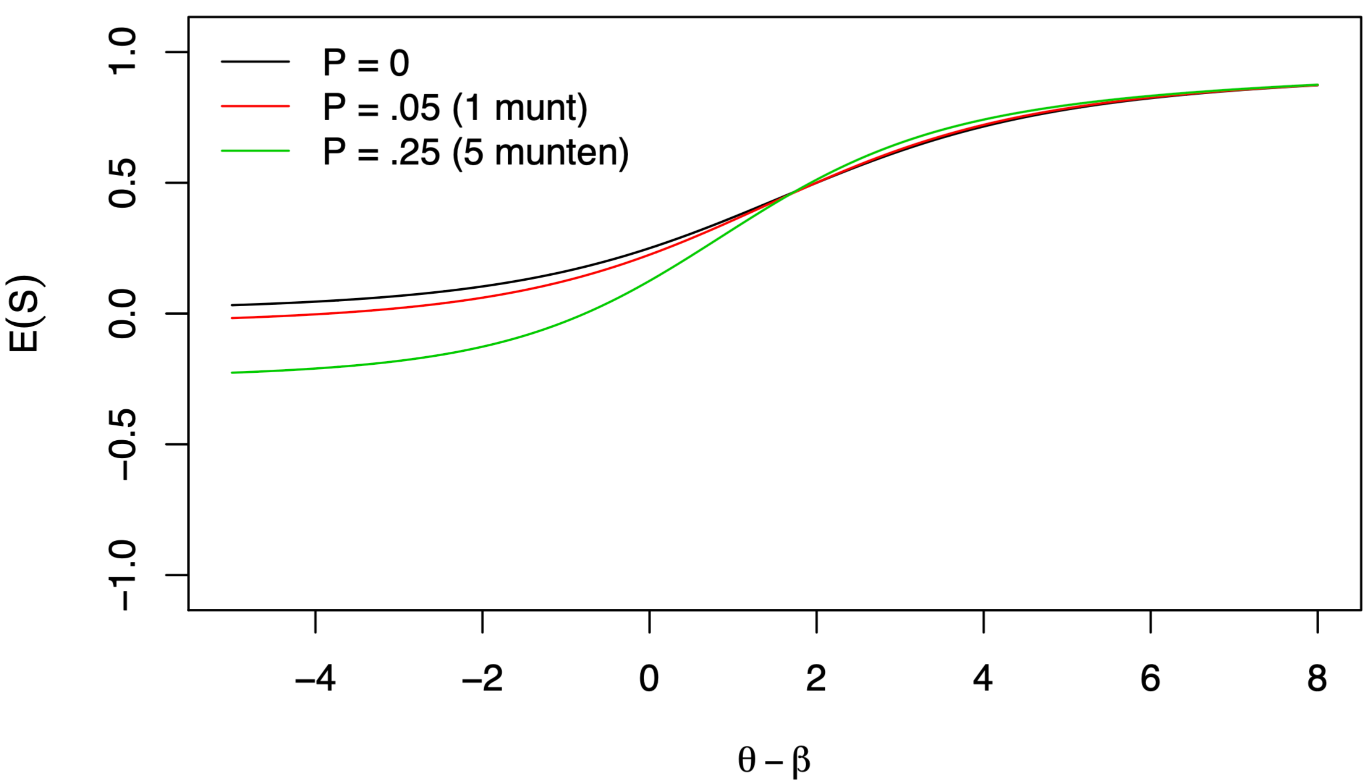
- Wide set of **games** (mainly focused on primary school)

- We **track the abilities estimates** using an adaptive elo algorithm (Klinkenberg [7]) for each game and an extended measurement model including **accuracy** and **time** (Maris [8])



Intermezzo | *Why Psychometrics?*





$$\theta_p^{t+1} = \theta_p^t + \mathcal{K}(S - \mathbb{E}(S_{ip}))\,,$$
$$\delta_i^{t+1} = \delta_i^t - \mathcal{K}(S - \mathbb{E}(S_{ip}))\,,$$

Methods | *Data: Math Garden*

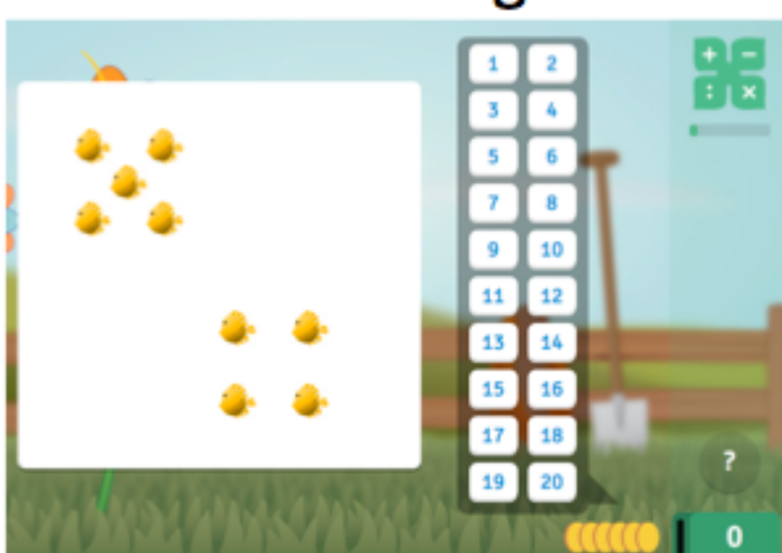
Data Selection

- Two large data sets:
  (1) Counting and Addition (N = 11.980)
  (2) Multiplication and Division (N =12.368)

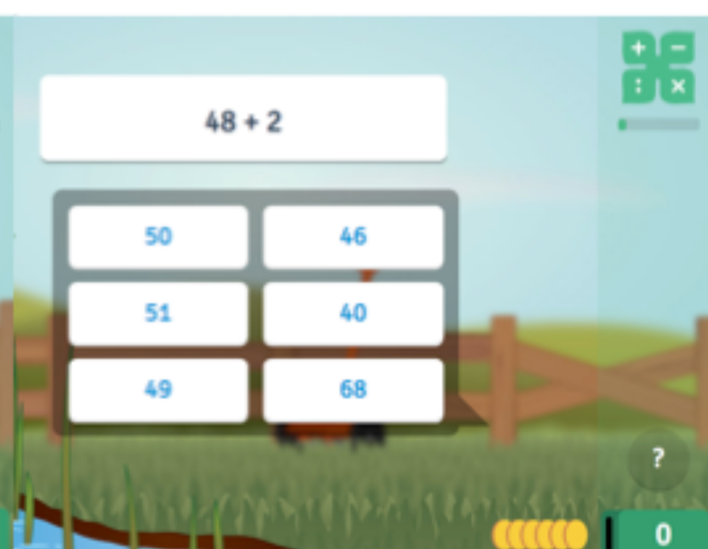
- Three time-points:
  T0 = Sep (start school year)
  T1 = Jan (middle)
  T2 = May-June (end)

- Included if subject played at least both domains once (missing data -> Full Information Maximum Likelihood)

Counting

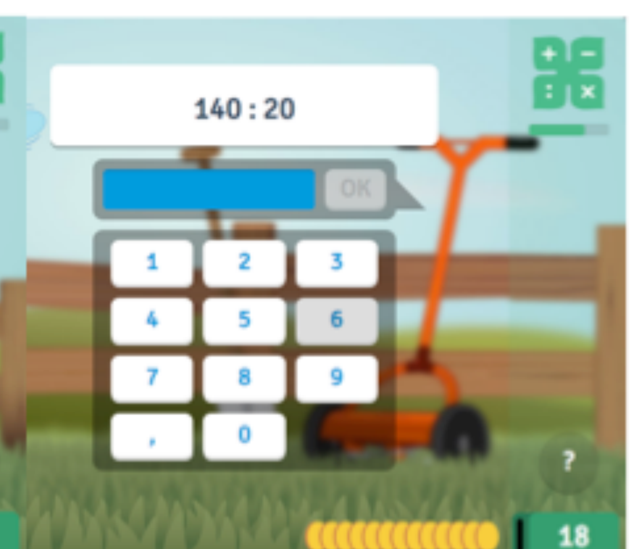


Addition



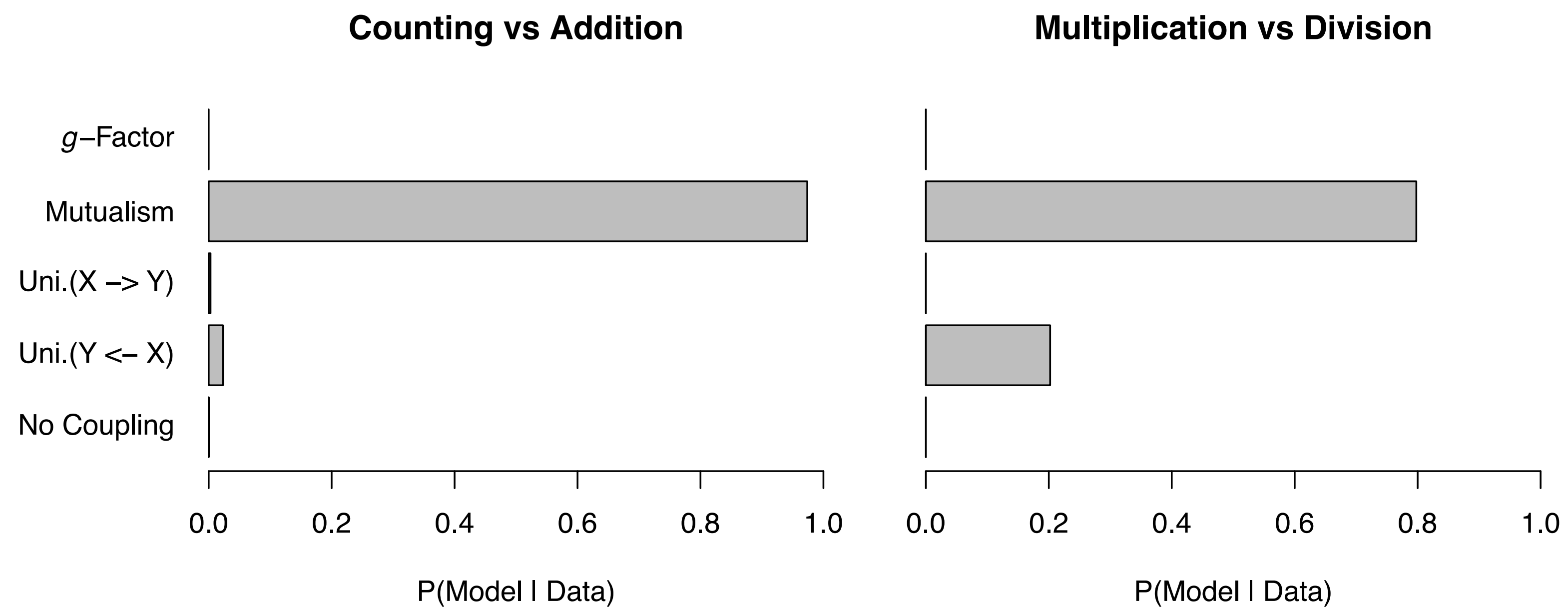
Multiplication

Division



Results | *Model Comparison*

AIC-weights:



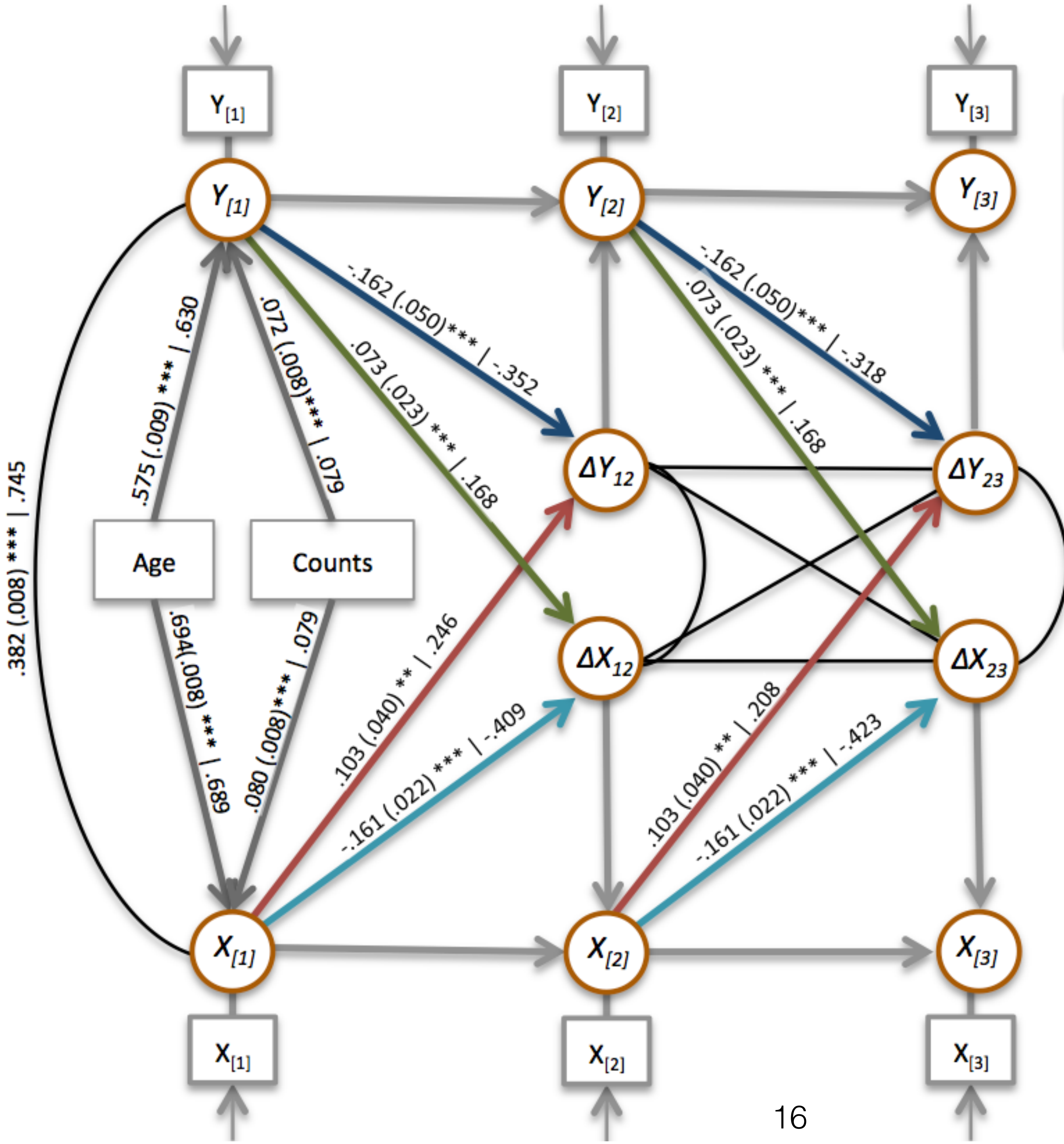
Fit statistics:

Table 2: Fit statistics for the different LSCM estimated on both datasets.

Domains	Model	Chi	df	CFI	RMSEA	SRMR	AIC	BIC
Counting	*g*-Factor	571.66	15	0.987	0.056	0.046	142138	142352
&	**Mutualism**	461.48	11	0.989	0.059	0.038	142036	142279
Addition	Uni.(C $\rightarrow$ A)	475.05	12	0.989	0.057	0.039	142048	142284
	Uni.(C $\leftarrow$ A)	470.95	12	0.989	0.057	0.038	142044	142279
	No coupling	485.24	13	0.989	0.056	0.038	142056	142284
Multipli-	*g*-Factor	671.52	15	0.989	0.059	0.030	149304	149520
cation	**Mutualism**	517.17	11	0.991	0.061	0.026	149158	149403
&	Uni.(M $\rightarrow$ D)	575.44	12	0.990	0.062	0.027	149214	149452
Division	Uni.(M $\leftarrow$ D)	521.91	12	0.991	0.059	0.026	149161	149398
	No coupling	597.61	13	0.990	0.060	0.026	149234	149464

Note. The the best fitting models are printed in bold. Uni. = Unidirectional Model

Results | *Model Parameters (1)*



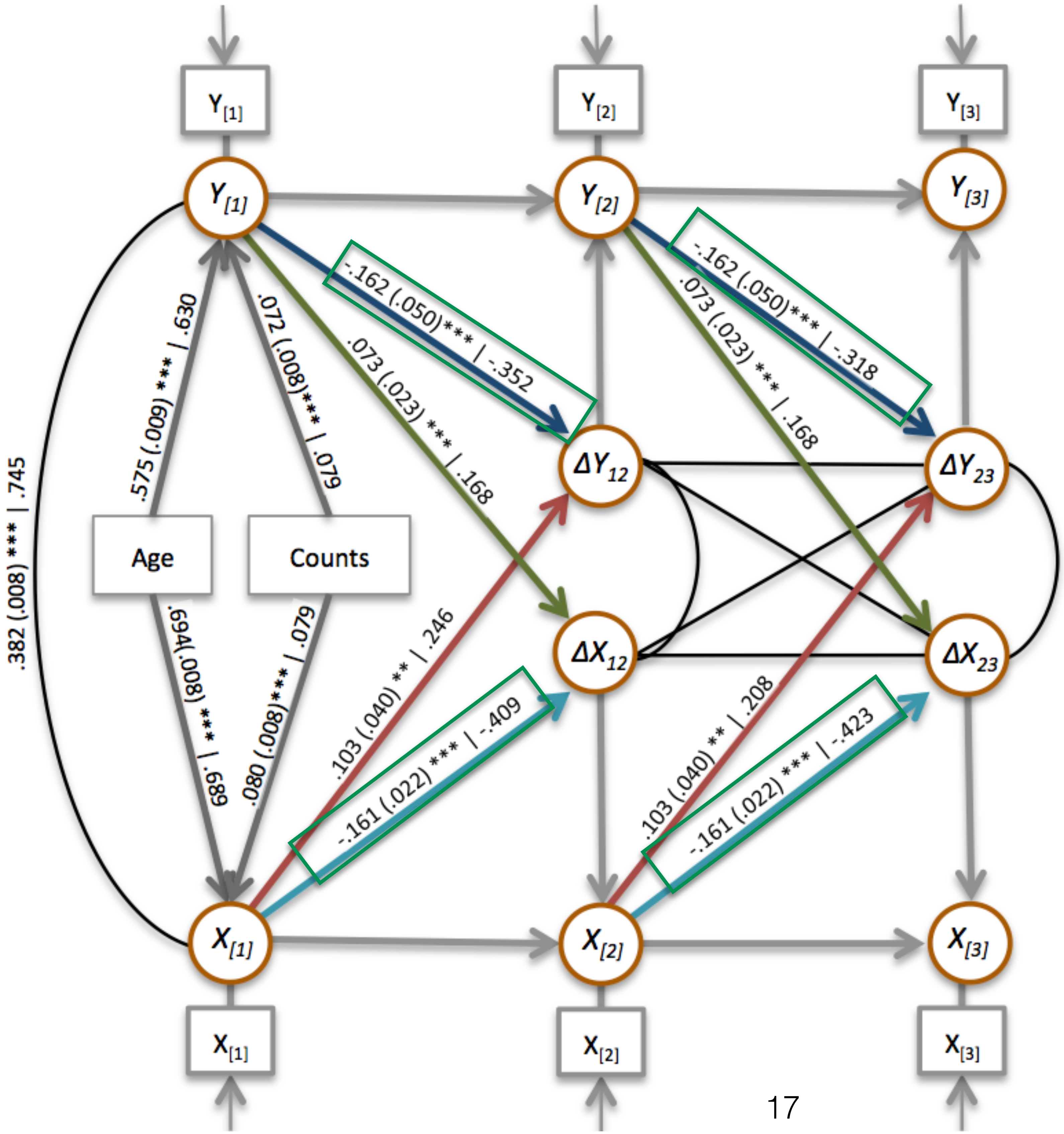
Means:
$X_{[1]}$ = -.084
ΔX_{12} = .339 (.006)***
ΔX_{23} = .267 (.006)***
$Y_{[1]}$ = -.032
ΔY_{12} = .354 (.008)***
ΔY_{23} = .164 (.009) ***

Correlations Δ:
$COR(\Delta X_{12}, \Delta Y_{12})$ = .093 (.006) *** | .546***
$COR(\Delta X_{23}, \Delta Y_{23})$ = .089 (.005) *** | .566***
$COR(\Delta X_{12}, \Delta X_{23})$ = -.005 (.008) | -.037
$COR(\Delta Y_{12}, \Delta Y_{23})$ = -.009 (.012) | -.044
$COR(\Delta X_{12}, \Delta Y_{23})$ = -.002 (.007) | -.011
$COR(\Delta Y_{12}, \Delta X_{23})$ = -.008 (.007) | -.054

Variances:
$X_{[1]}$ = .528 (.011) *** | .520
ΔX_{12} = .145 (.013) *** | .922
ΔX_{23} = .128 (.017) *** | .916
$Y_{[1]}$ = .498 (.018) *** | .597
ΔY_{12} = .171 (.026) *** | .964
ΔY_{23} = .225 (.035) *** | .957

$X_{[1-3]}$ = .034 (.008) ***
$Y_{[1-3]}$ = .106 (.018) ***

Results | *Model Parameters (2)*



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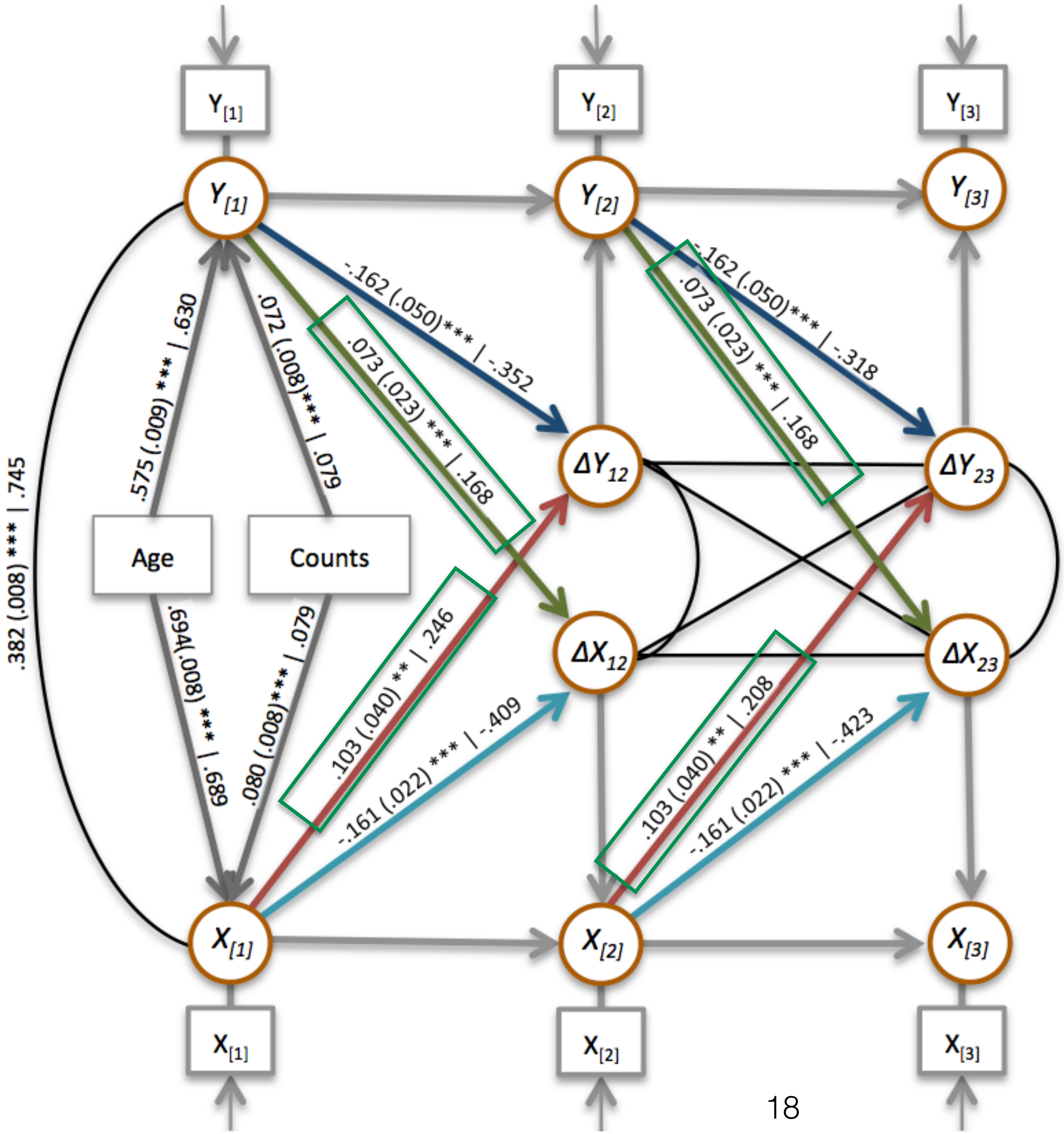
Results | *Model Parameters (3)*



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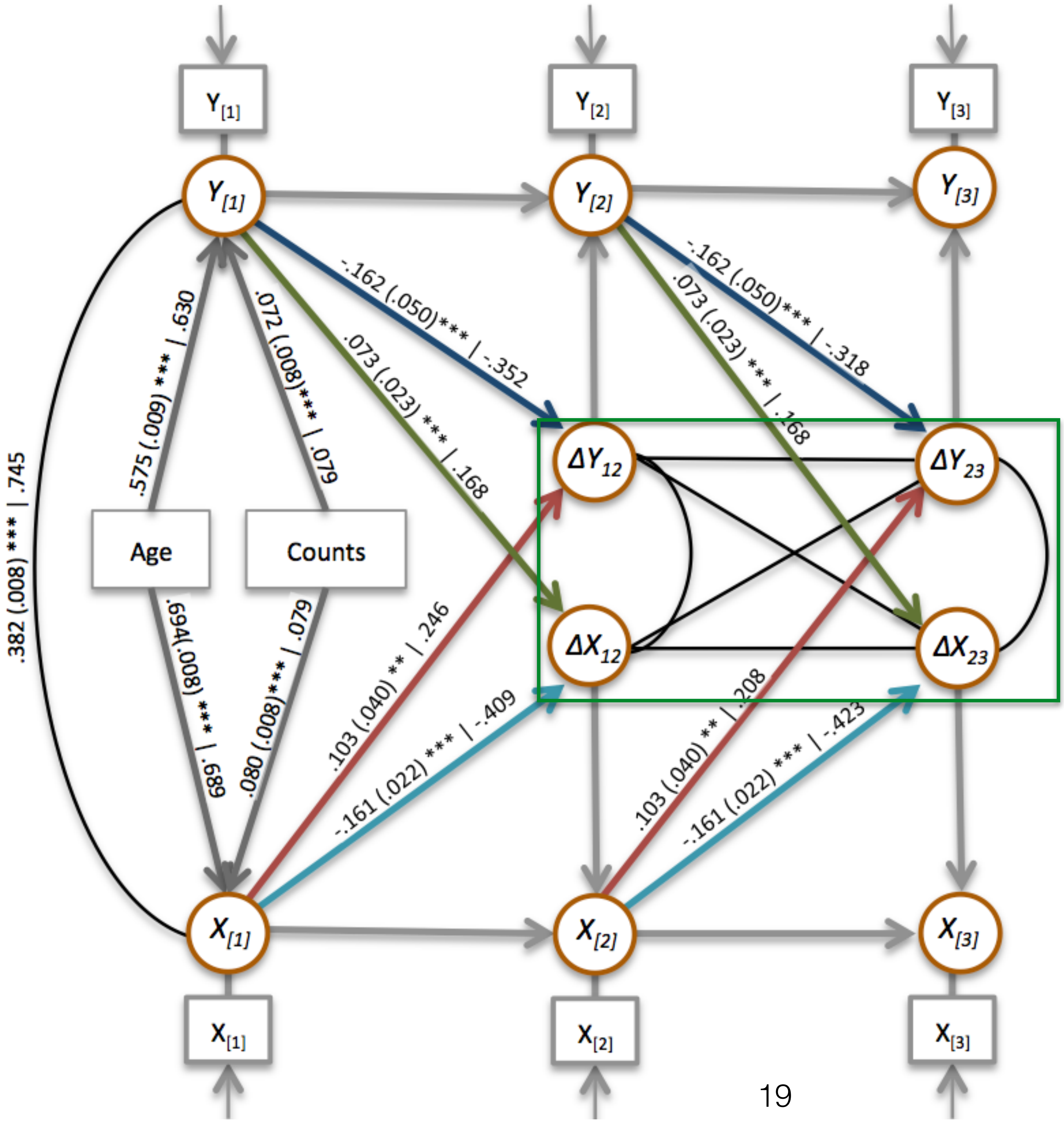
Results | *Model Parameters (4)*



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Conclusion | *G vs Mutualism*

- Positive manifold, can be explained by mutualistic effects not present according to a *g*-factor account of cognitive development (replicating the results of Kievit [2017])

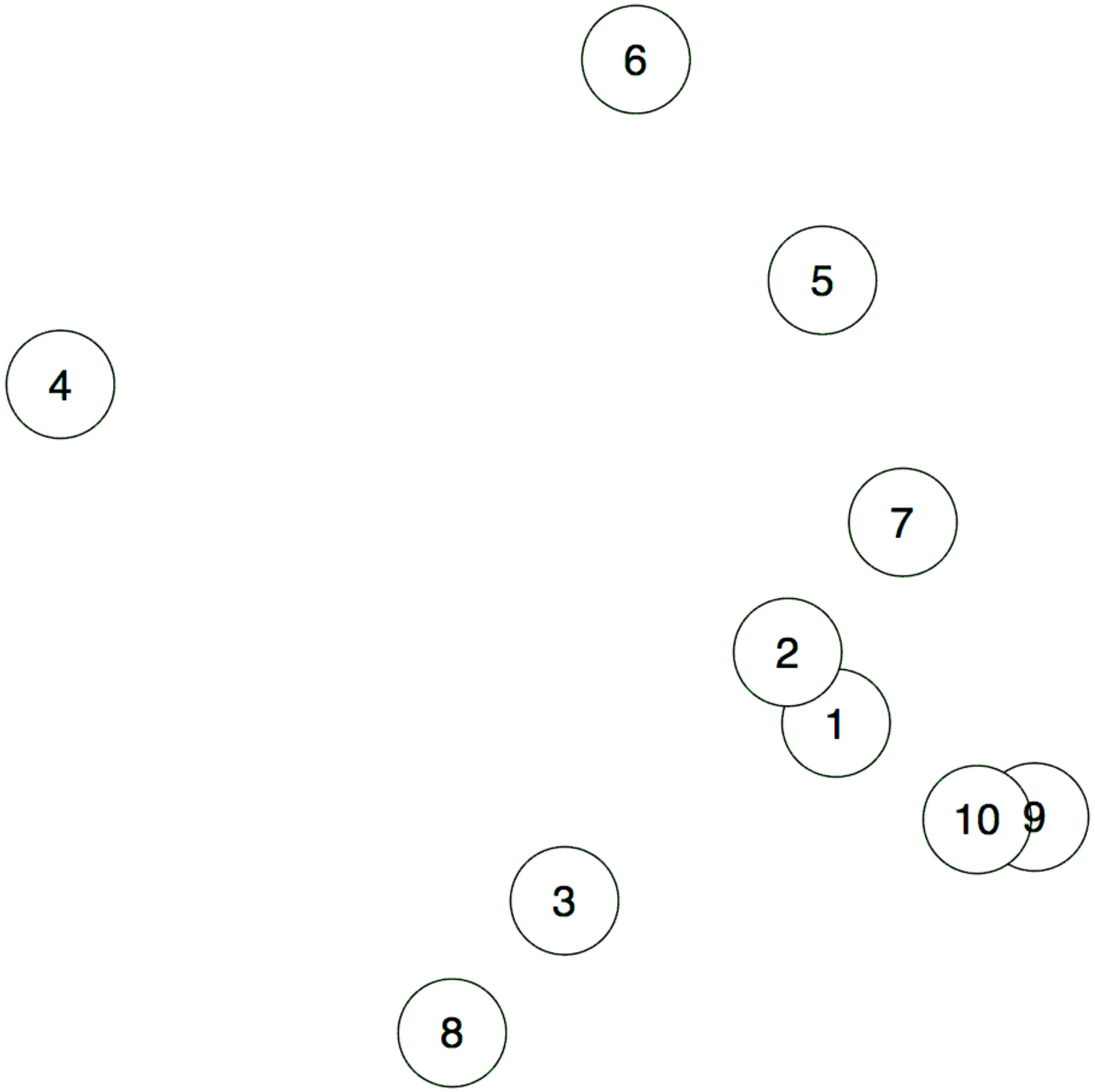
- Significant coupling is needed for an accurate description of the data.

- Hybrid account: the remaining correlational structure between change scores could both be explained by *g* and *mutualistic* effects (data selection effects)

Discussion | *G vs Mutualism*

- Mechanisms of coupling?
- Mutualism at what level: abilities (factors) or item responses? (Wired Cognition, with Alexander Savi, Gunter Maris & Han van der Maas)
- Model extensions: more variables, time-points & individual differences in coupling.
- Replications: smaller experimental dataset & large data sets with other domains.
- Positive manifold is everywhere (intelligence data; scholastic abilities; depression; …)

Developing Correlations | *G vs Mutualism*



Discussion | *Why Psychometrics?*

- G-factor -> Psychometrics
- Mutualism = Data Generating Model (and not true)
- Mutualism != Cognitive Model (Architecture)
- The field of psychometrics is evolved around individual differences. Cognition looks for similarities.

References | *Questions*

[1] Borsboom, Mellenbergh & Van Heerden (2003). The theoretical status of latent variables. Psychological review, 110, 203.

[2] Van der Maas et al (2006) A dynamical model of general intelligence: the positive manifold of intelligence by mutualism. Psychological review,113, 842–861.

[3] Kruis & Maris (2016) Three representations of the ising model. Scientific Reports, 6, 1–11

[4] Spearman (1927). The abilities of man. Macmillan

[5] McArdle (2009). Latent variable modeling of differences and changes with longitudinal data. Annual review of psychology, 60, 577–605.

[6] Klinkenberg, Straatemeier & Van der Maas (2011) Computer adaptive practice of maths ability using a new item response model for on the fly ability and difficulty estimation. Computers & Education, 57, 1813–1824.

[7] Maris & Van der Maas (2012). Speed-accuracy response models: Scoring rules based on response time and accuracy. Psychometrika,77, 615–633.

[8] Kievit et al (2017). Mutualistic coupling between vocabulary and reasoning supports cognitive development during late adolescence and early adulthood. Psychological Science.

Link to OSF

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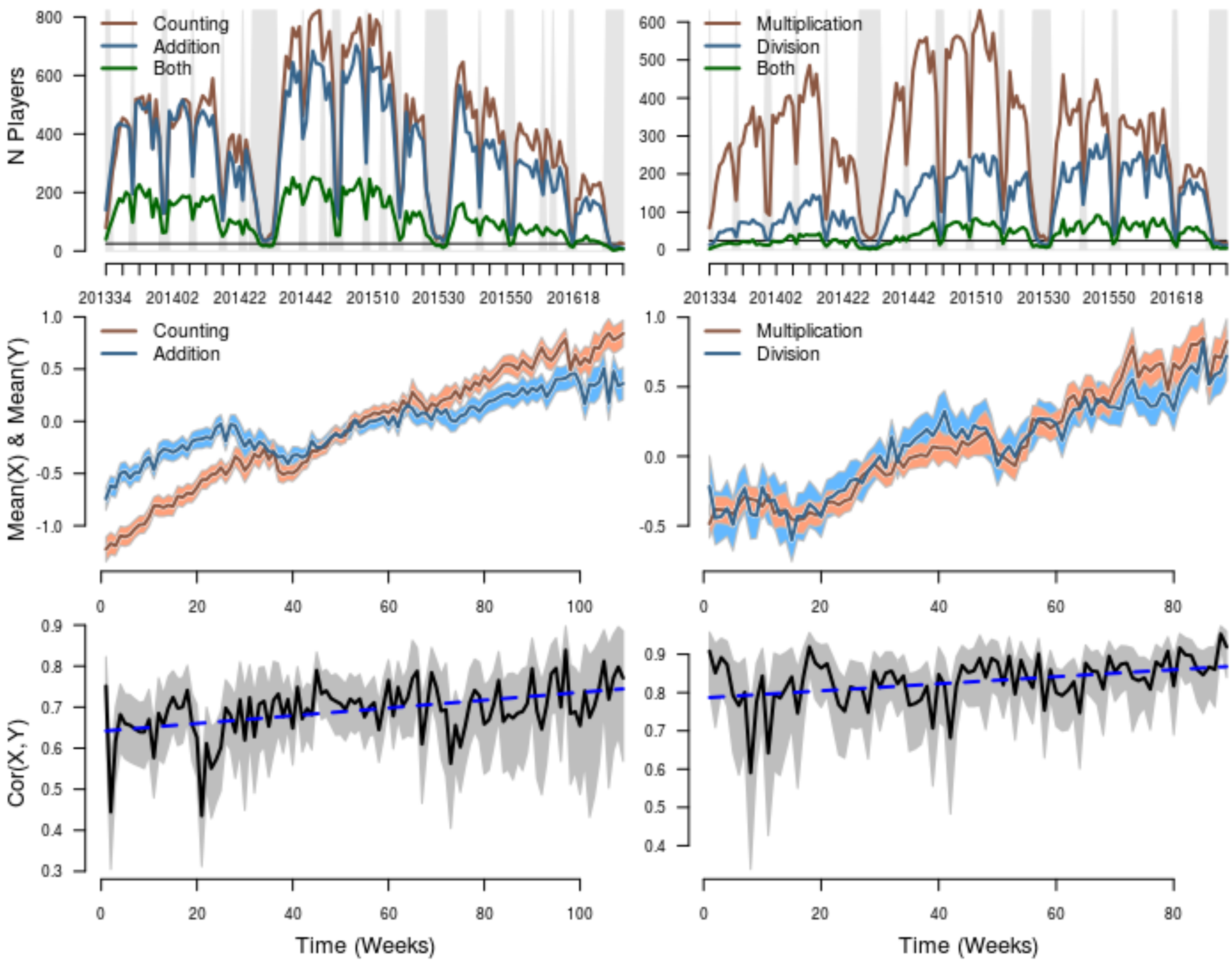
Developing Correlations | *G vs Mutualism*



Simulations | *Model Comparison*

- True model (coupling = 0):
  (g-factor = co-coupling) > mutualism

- True model (coupling > 0.1):
  mutualism > g-factor > co-coupling